Novel Simplified Time-Periodic Explicit Error Correction Method for Steady-State Analysis of Magnetic Field Including Direct Current Component

Hirokatsu Katagiri, Yoshihiro Kawase and Tadashi Yamaguchi
Department of Information Science, Gifu University
1-1, Yanagido, Gifu, 501-1193, Japan
E-mail khiro@fem.info.gifu-u.ac.jp

Abstract — In order to obtain the steady-state of the time-periodic magnetic field including the direct current (DC) component rapidly, we develop a novel simplified time periodic-explicit error correction (TP-EEC) method. The usefulness of our novel method is verified through the steady-state eddy-current analysis of the IEEJ standard benchmark model using the three-dimensional finite element method (3-D FEM).

I. INTRODUCTION

In steady-state analyses of the time-periodic magnetic field taking into account the eddy current using the step-by-step time integration method, the numerical transient state appears in the time steps. Therefore, many time step calculations are required to obtain the steady state. In order to improve the convergence characteristics for the steady-state analysis, the simplified time periodic-explicit error correction (TP-EEC) method is developed [1]. In the method, the numerical transient can be extracted by using the half cycle periodicity of the magnetic vector potentials. Therefore, the method is applied to only the steady-state analysis of electromagnetic equipments, which have the magnetic field with the half cycle periodicity.

In the fields of the industrial application, there are many electromagnetic apparatus, which have the direct current (DC) magnetic field superposed on the alternating current (AC) magnetic field, like permanent motors. The conventional simplified TP-EEC method can not be applied to them, because the magnetic field does not satisfy the half cycle periodicity.

In this paper, we develop a novel simplified TP-EEC method to improve the convergence characteristics for the steady-state analysis of the magnetic field including the DC component. In our novel method, the half cycle periodicity of the eddy current density $\boldsymbol{J}_e$ is employed. $\boldsymbol{J}_e$ does not include the DC component even if the magnetic vector potential $\boldsymbol{A}$ includes the DC component, because $\boldsymbol{J}_e$ can be expressed as the time derivative of $\boldsymbol{A}$. The usefulness of our novel method is verified through the steady-state eddy-current analysis of the IEEJ standard benchmark model using the three-dimensional finite element method (3-D FEM) [2].

II. NOVEL SIMPLIFIED TP-EEC METHOD

The fundamental equations of the magnetic field can be written using the magnetic vector potential $\boldsymbol{A}$ as follows:

$$\mathrm{rot}(\nu\,\mathrm{rot}\,\boldsymbol{A}) = \boldsymbol{J}_0 + \boldsymbol{J}_e + \nu_0\,\mathrm{rot}\,\boldsymbol{M} \qquad (1)$$

$$\boldsymbol{J}_e = -\sigma \frac{\partial \boldsymbol{A}}{\partial t} \qquad (2)$$

where ν is the reluctivity, $\boldsymbol{J}_0$ is the exciting current density, $\boldsymbol{J}_e$ is the eddy current density, ν_0 is the reluctivity of vacuum, $\boldsymbol{M}$ is the magnetization and σ is the conductivity.

The conventional simplified TP-EEC method uses the half cycle periodicity of $\boldsymbol{A}$ to improve the convergence characteristics for the steady-state analysis. Therefore, the convergence characteristics for the steady-state analysis of the magnetic field including the DC component are not improved by the conventional TP-EEC method.

In our novel method, the half cycle periodicity of the eddy current density $\boldsymbol{J}_e$ is employed. $\boldsymbol{J}_e$ does not include the DC component even if the magnetic vector potential $\boldsymbol{A}$ includes the DC component, because $\boldsymbol{J}_e$ can be expressed as the time derivative of $\boldsymbol{A}$. Consequently, $\boldsymbol{J}_e$ satisfies the half cycle periodicity as follows:

$$\boldsymbol{J}_e(t) = -\boldsymbol{J}_e(t - T/2) \qquad \text{(at steady state)} \qquad (3)$$

where T is the time period of $\boldsymbol{J}_e$.

By using the half cycle periodicity of $\boldsymbol{J}_e$, $\boldsymbol{J}_e$ can be corrected like the conventional simplified TP-EEC method as follows:

$$\widetilde{\boldsymbol{J}}_e = \frac{\boldsymbol{J}_e(t) - \boldsymbol{J}_e(t - T/2)}{2} \qquad (4)$$

where $\widetilde{\boldsymbol{J}}_e$ is the modified eddy current density.

By using $\widetilde{\boldsymbol{J}}_e$, (1) can be written as follows:

$$\mathrm{rot}(\nu\,\mathrm{rot}\,\widetilde{\boldsymbol{A}}\,) = \boldsymbol{J}_0 + \widetilde{\boldsymbol{J}}_e + \nu_0\,\mathrm{rot}\,\boldsymbol{M} \qquad (5)$$

where $\widetilde{\boldsymbol{A}}$ is the modified magnetic vector potential by using the half cycle periodicity of $\boldsymbol{J}_e$.

III. STEADY-STATE EDDY-CURRENT ANALYSIS OF IEEJ STANDARD BENCHMARK MODEL

To verify the usefulness of our novel method, the steady-state eddy-current of the IEEJ standard benchmark model [2] is analyzed using the 3-D FEM. Fig. 1 shows the IEEE standard benchmark model. The conductivity of the aluminum plate is 31,250,000 S/m and the relative permeability of the ferrite core is 3,000. Because of the symmetry, the analyzed region is 1/8 of whole model. Fig. 2 shows the input current waveform, which is the sinusoidal waveform including the DC component, to verify our novel method.

The correction of $\boldsymbol{J}_e$ by our novel method or $\boldsymbol{A}$ by the conventional TP-EEC method are applied at 189°, 369° and 549° of the electrical angles.

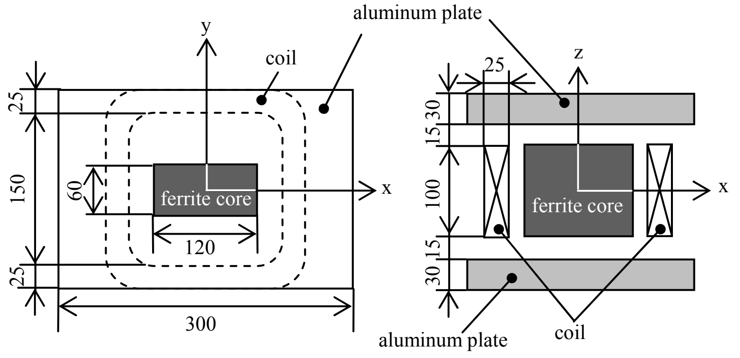
Fig. 3 shows the error of the instantaneous eddy current loss in the aluminum plate, which are normalized by the eddy current loss at the steady state. The error means the difference between the instantaneous value and the value at the steady state.

The conventional simplified TP-EEC method makes the convergence characteristics worse than that without the simplified TP-EEC method.

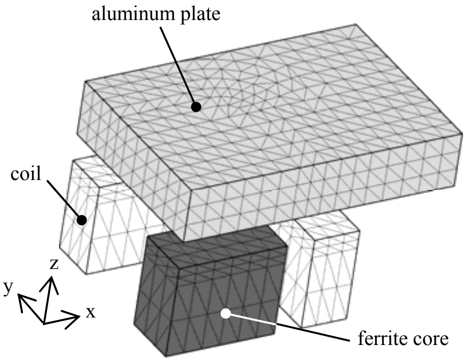
In our novel method, though the convergence characteristic becomes worse immediately after the correction, after that it becomes extremely better.

Table I shows the discretization data and CPU time. The time step calculations are repeated until the error of the instantaneous eddy current loss becomes less than 1%. The CPU time with our novel method is approximately 1/3 of that without the simplified TP-EEC method.

In full paper, our novel method will be applied to the steady-state analysis of practical models and the results will be also shown.



(a) plane view (b) front view



(c) mesh for the analysis (1/8 of whole model)

Fig. 1. IEEJ standard benchmark model of three-dimensional eddy-current analysis.

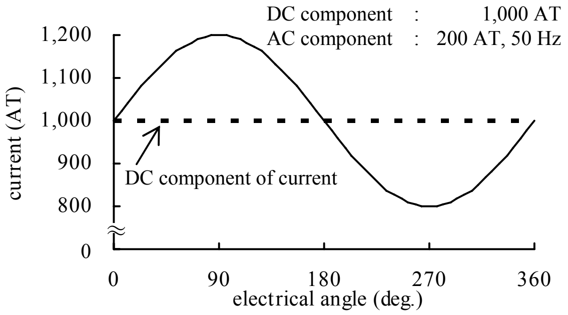


Fig. 2. Input current waveform.

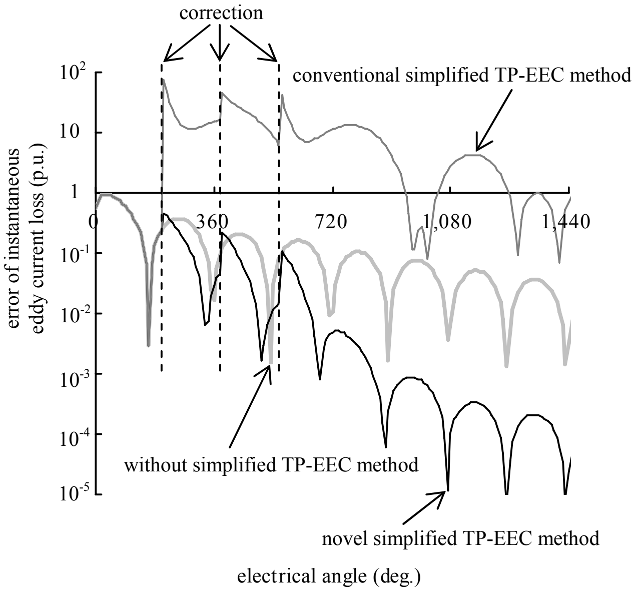


Fig. 3. Error of instantaneous eddy current loss.

TABLE I DISCRETIZATION DATA AND CPU TIME

	Without simplified TP-EEC method	Novel simplified TP-EEC method
Number of elements	33,600	
Required time steps for steady states	214	73
CPU time (s)	193	71

Computer used: Intel Core2 Duo (2.66GHz) PC

IV. REFERENCES

[1] Y. Takahashi, T. Tokumasu, A. Kameari, H. Kaimori, M. Fujita, T. Iwashita and S. Wakao, “Convergence Acceleration of Time-Periodic Electromagnetic Field Analysis by Singularity Decomposition-Explicit Error Correction Method,” *IEEE Trans. Magn*. vol. 46, no. 8, pp.2947-2950, 2010.

[2] T. Nakata, N. Takahashi, T. Imai and K. Muramatsu, “Comparison of Various Methods of Analysis and Finite Elements in 3-D Magnetic Field Analysis,” *IEEE Trans. Magn*. vol. 27, no. 5, pp.4073-4076, 1991.